FPT results through potential maximal cliques

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in collaboration with

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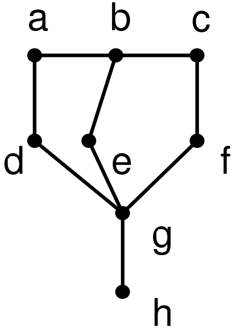
October 15, 2015

Minimal separators



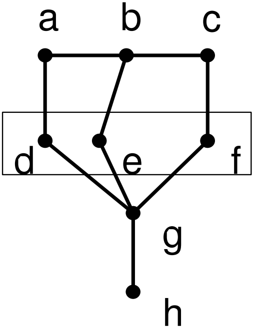
Definition

$S \subseteq V$ is a minimal a, b-separator if S separates a and b and it is inclusion-minimal for this property.

Definition

S is a minimal separator of G there exist vertices a, b s.t. S is a minimal a, b-separator.

Minimal separators



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Potential maximal cliques

Definition

A set of vertices Ω is a potential maximal clique of G if is a maximal clique in some minimal triangulation of G.

Proposition (Bouchitté, Todinca 2001)

The number of potential maximal cliques is polynomial in the number of minimal separators.

Minimal separators and $\mathcal{G}_{poly}$

Definition

For a polynomial *poly*, let $\mathcal{G}_{poly}$ be the class of graphs such that $G \in \mathcal{G}_{poly}$ if G has at most $poly(n)$ minimal separators.

Class	**minimal separators**	**potential maximal cliques**
Weakly chordal	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$
Chordal	$\mathcal{O}(n)$	$\mathcal{O}(n)$
Polygon-circle	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$
Circle	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$
Circular arc	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$
d-trapezoid	$\mathcal{O}(n^d)$	$\mathcal{O}(n^{d+2})$

When the input graph belongs to $\mathcal{G}_{poly}$ we can find in polynomial time a MAXIMUM INDUCED:

- INDEPENDENT SET, FOREST, PATH, MATCHING
- SUBGRAPH WITH NO CYCLES $\geq l$, OUTERPLANAR
- SUBGRAPH WITH NO CYCLES OF LENGTH $0 \mod l$

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For $t \geq 0$ and $\mathcal{P}$ a CMSO property:

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t

Input: A graph $G = (V, E)$

Output A largest induced subgraph $G[F]$ s.t.

- $tw(G[F]) \leq t$,
- $G[F]$ satisfies $\mathcal{P}$.

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Theorem (Fomin, Todinca, Villanger, 2015)

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t ON $\mathcal{G}_{poly}$ *is solvable in polynomial time.*

When the input graph belongs to $\mathcal{G}_{poly}$ we can find in polynomial time a MAXIMUM INDUCED:

- INDEPENDENT SET, FOREST, PATH, MATCHING
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Theorem (Fomin, Todinca, Villanger, 2015)

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t *is solvable time* $\mathcal{O}(\#\text{POTENTIAL MAXIMAL CLIQUES} \cdot n^{t+cst} \cdot f(\mathcal{P}, t))$.

In this talk, two extensions:

1) **Theorem** (Fomin, Liedloff, M., Todinca. SWAT 2014)

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t *can be solved in time* $\mathcal{O}^*(4^{\mathrm{vc}})$ *and* $\mathcal{O}^*(1.7347^{\mathrm{mw}})$.

2) **Definition** ($\mathcal{G}_{poly} + kv$)

$G \in \mathcal{G}_{poly} + kv$ if there exists a set $M \subset V$ called modulator, $|M| \leq k$ s.t. $G - M \in \mathcal{G}_{poly}$.

Theorem (Liedloff, M. and Todinca. WG 2015)

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t ON $\mathcal{G}_{poly} + kv$ *with parameter k is fixed-parameter tractable, when the modulator is also part of the input.*

First result

Theorem (Fomin, Todinca, Villanger, 2015)

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t *is solvable time* $\mathcal{O}(\#\text{POTENTIAL MAXIMAL CLIQUES} \cdot n^{t+cst} \cdot f(\mathcal{P}, t))$.

Our result:

Theorem

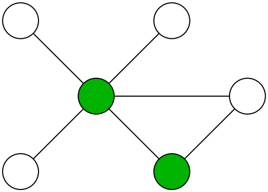
The number of potential maximal cliques is bounded by

- $\mathcal{O}^*(4^{vc})$
- $\mathcal{O}^*(1.7347^{\mathrm{mw}})$

Vertex cover

Definition

The vertex cover of a graph G, denoted by vc(G), is the minimum number of vertices that cover all edges of the graph.



- The number of minimal separators is bounded by 3^{vc}
- The number of potential maximal cliques is bounded by $\mathcal{O}^*(4^{vc})$

Vertex cover and minimal separators

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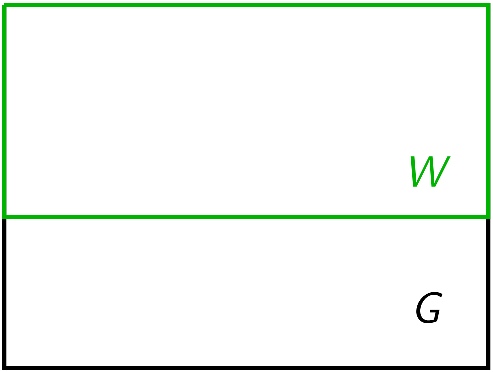

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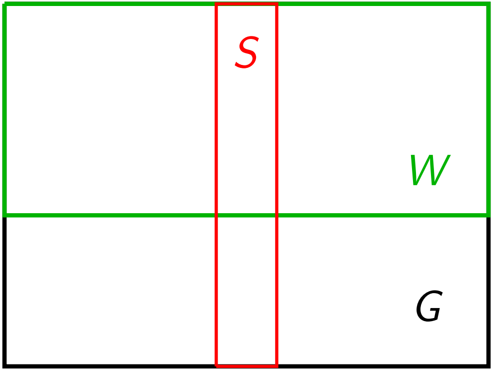

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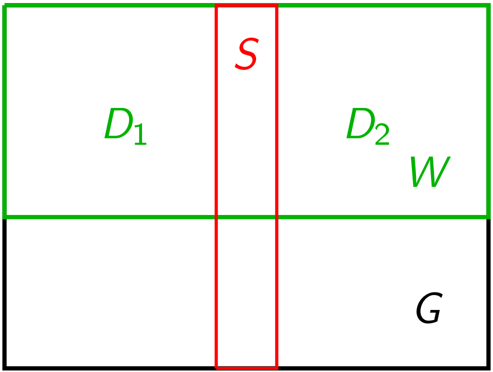

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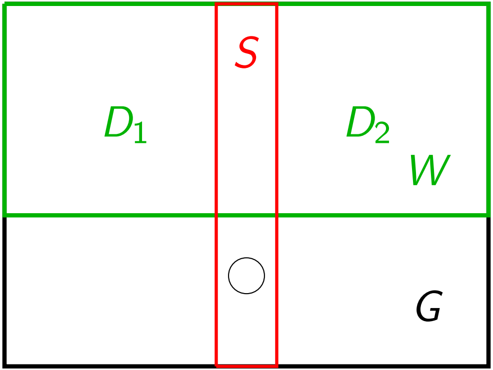

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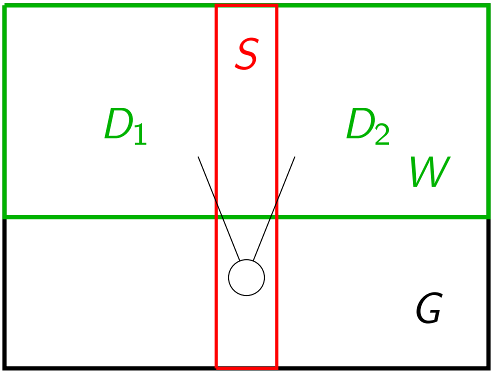

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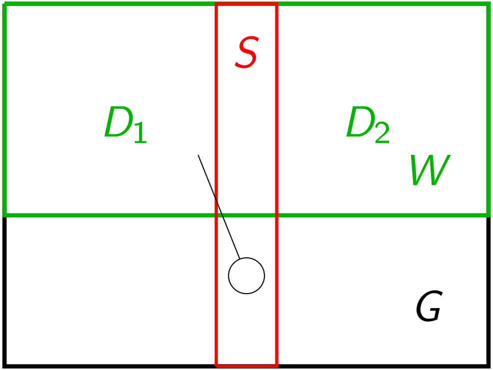

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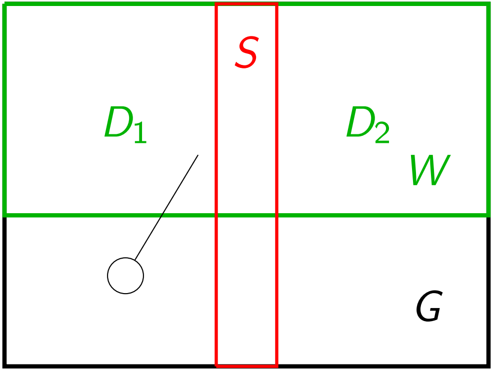

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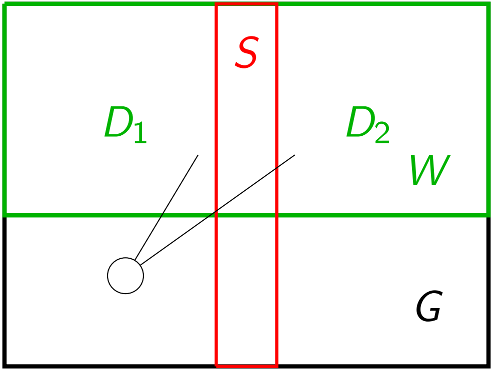

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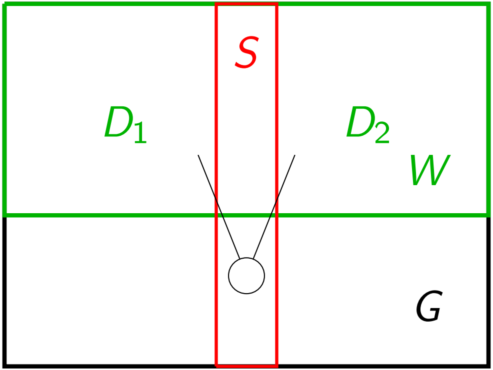

Vertex cover and minimal separators

For any vertex cover W

$$S \to (S^W, D_1, D_2) = W$$

$$S = S^W \cup \{x \in V \setminus W \mid N(x) \text{ intersects both } D_1 \text{ and } D_2\}$$

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Theorem

- *Number of minimal separators is* $\mathcal{O}(3^{\mathrm{vc}})$
- *They can be listed in time* $\mathcal{O}^*(3^{\mathrm{vc}})$

Potential maximal cliques and minimal separators

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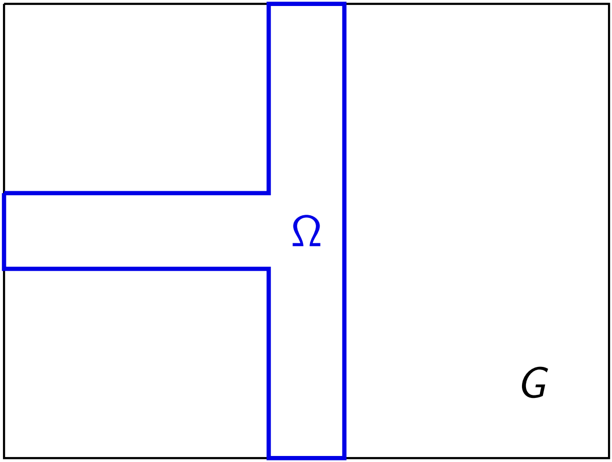

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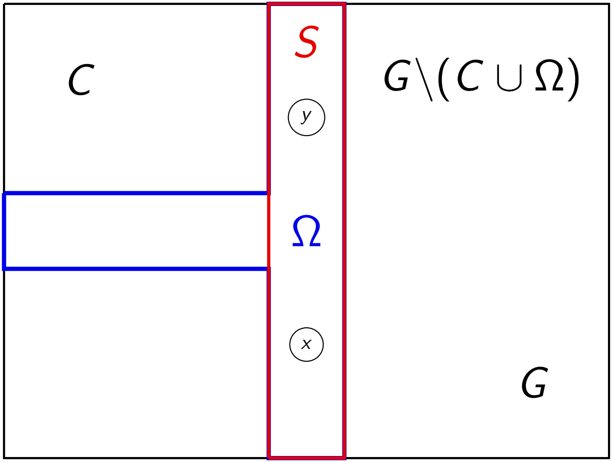

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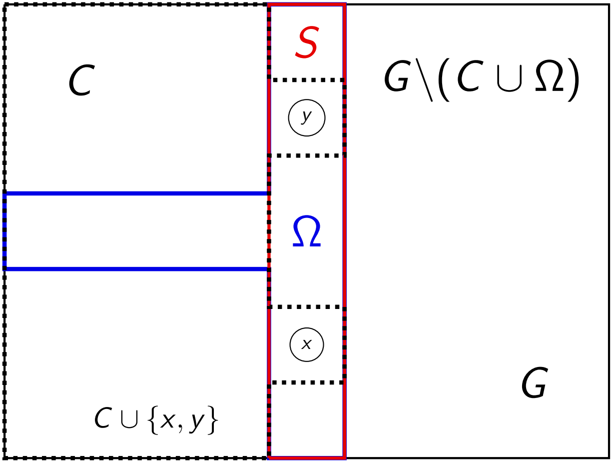

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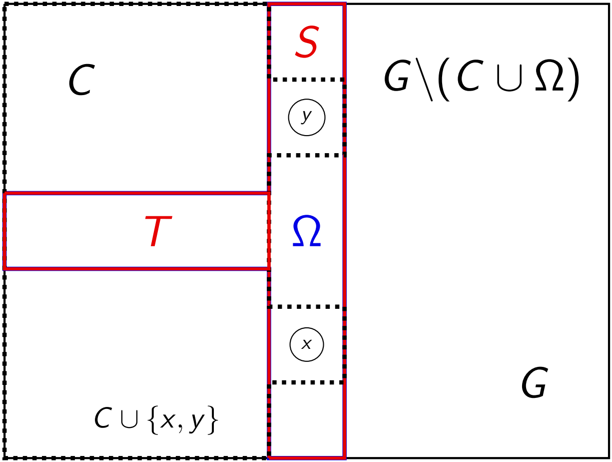
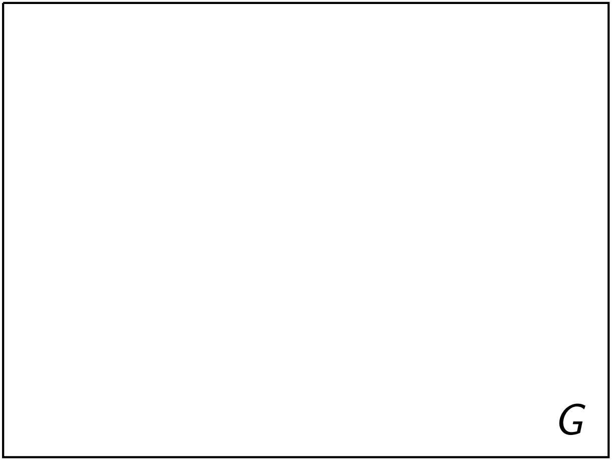


Vertex cover and pmc



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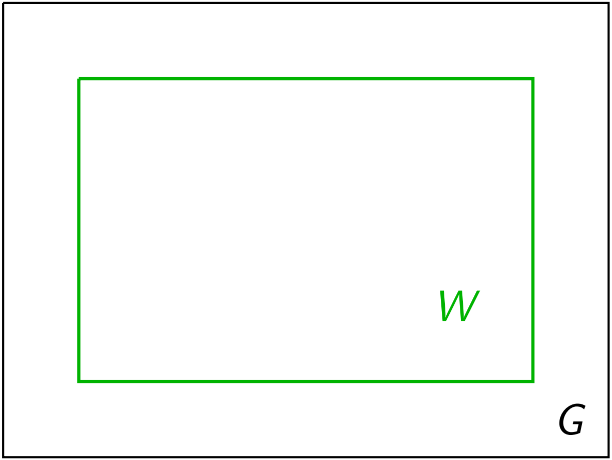

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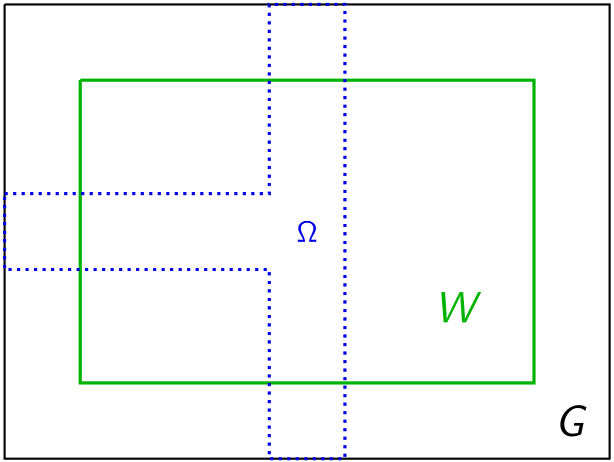

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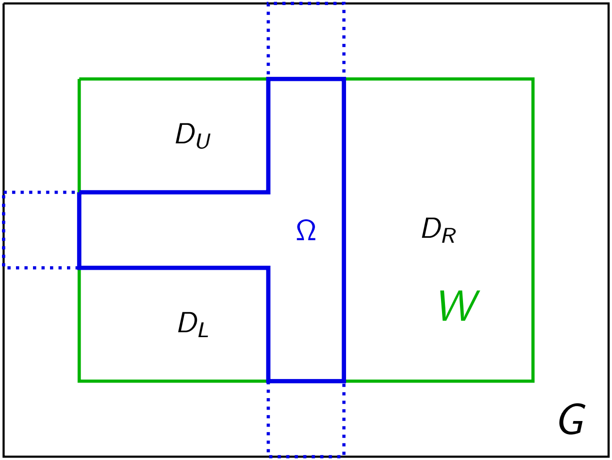

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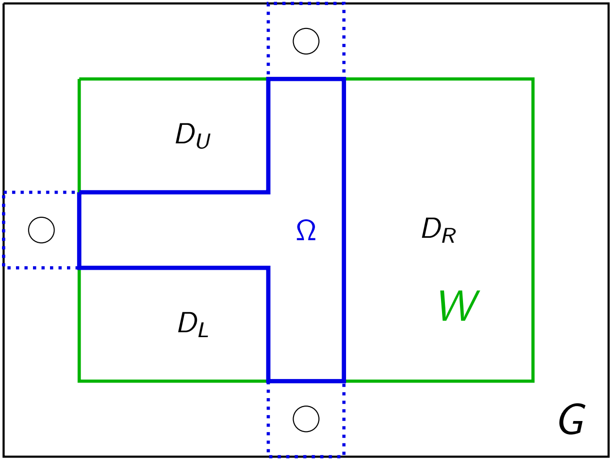

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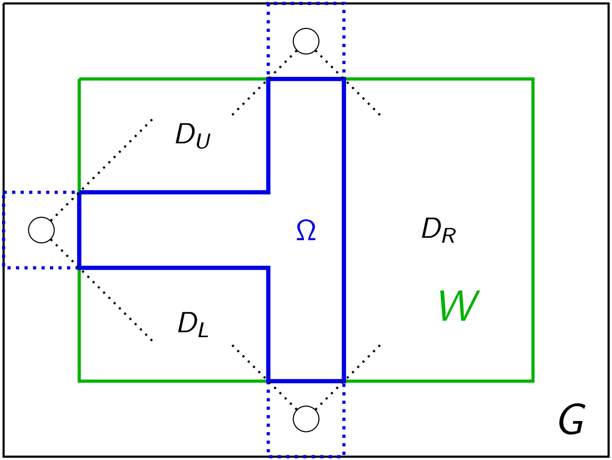

Vertex cover and pmc

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$$\Omega \to (\Omega^W, D_R, D_U, D_L) = W$$

$$\begin{aligned}\Omega &= \Omega^W \\ &\cup \{x \in V \setminus W \mid N(x) \text{ intersects both } D_R \text{ and } (D_U \cup D_L)\} \\ &\cup \{x \in V \setminus W \mid N(x) \text{ intersects both } D_U \text{ and } D_L\}\end{aligned}$$

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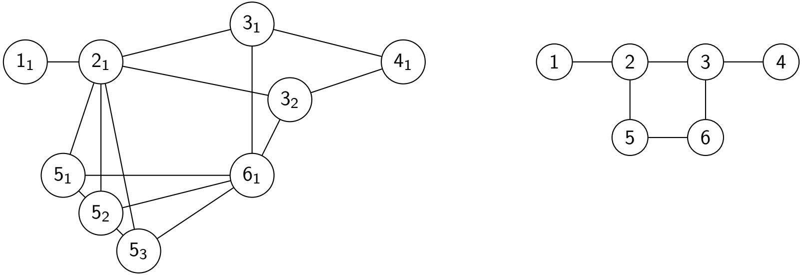
Theorem

- *Number of potential maximal cliques is* $\mathcal{O}^*(4^{\mathrm{vc}})$
- *They can be listed in time* $\mathcal{O}^*(4^{\mathrm{vc}})$

Modular width

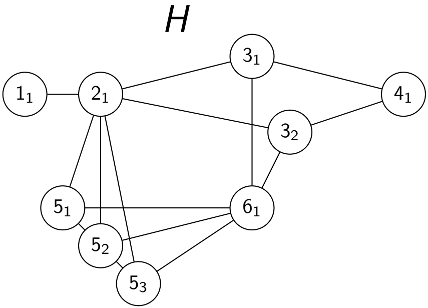
Definition

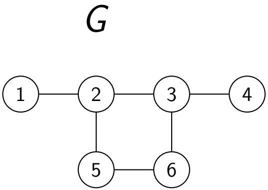
The modular width mw(G) can be defined as the maximum degree of a prime node in the modular decomposition tree of G



- The number of minimal separators is bounded by $\mathcal{O}^*(1.6181^{\mathrm{mw}})$,
- The number of potential maximal cliques is bounded by $\mathcal{O}^*(1.7347^{\mathrm{mw}})$.

Modular width and minimal separators





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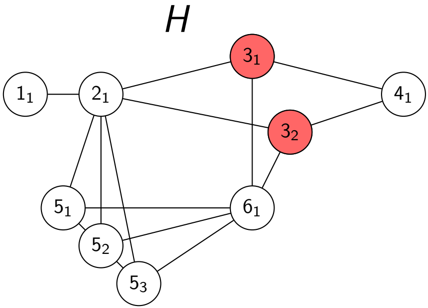

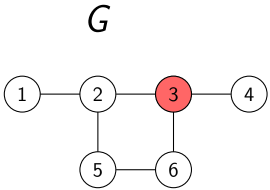

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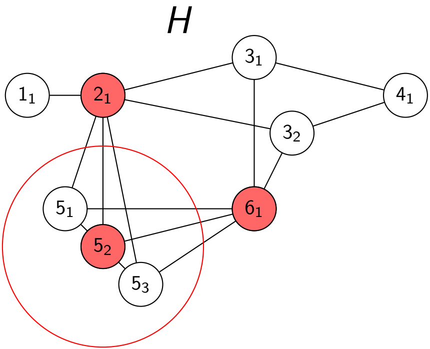

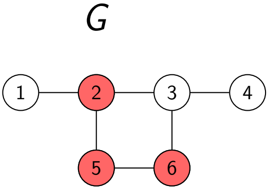

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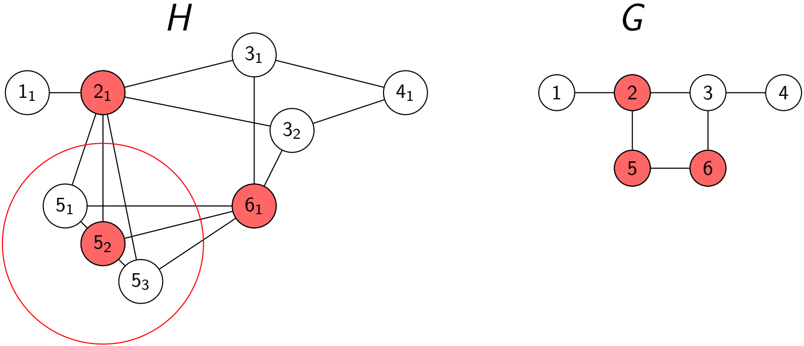

Theorem (Fomin, Villanger (2010))

Every n-vertex graph has $\mathcal{O}(1.6181^n)$ minimal separators and $\mathcal{O}(1.7347^n)$ potential maximal cliques.

Conclusion

Theorem

Optimal Induced Subgraph for $\mathcal{P}$ and t *can be solved in time* $\mathcal{O}^*(4^{\mathsf{vc}})$ *and* $\mathcal{O}^*(1.7347^{\mathsf{mw}})$

Conclusion

Theorem

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t *can be solved in time* $\mathcal{O}^*(4^{\mathsf{vc}})$ *and* $\mathcal{O}^*(1.7347^{\mathsf{mw}})$

Also..

- (Weighted) TREEWIDTH,
- (Weighted) MINIMUM FILL IN,
- TREELENGTH.

Are solvable in time $\mathcal{O}^*(\#\,\mathrm{pmc})$ ([Gysel], [Lokshtanov],..)

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OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND *t can be solved in time* $\mathcal{O}^*(4^{\mathsf{vc}})$ *and* $\mathcal{O}^*(1.7347^{\mathsf{mw}})$

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OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND *t can be solved in time $\mathcal{O}^*(4^{\mathsf{vc}})$ and $\mathcal{O}^*(1.7347^{\mathsf{mw}})$*

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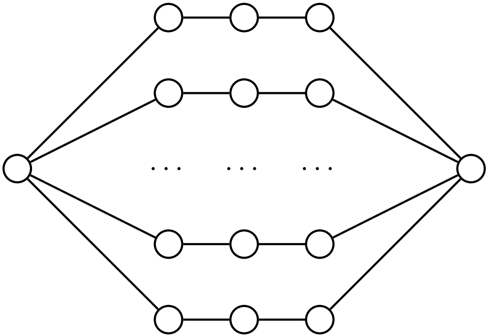
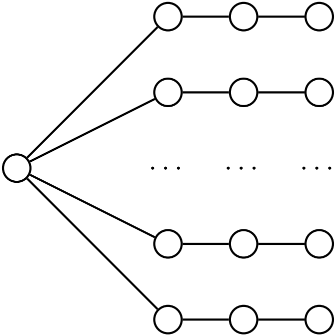
(then in time $\mathcal{O}^*(4^{\mathsf{vc}})$ and $\mathcal{O}^*(1.7347^{\mathsf{mw}})$).

- Running times that are single exponential in the parameter.
- This result covers both sparse and dense families of graphs.





Second result

Definition ($\mathcal{G}_{poly} + kv$)

$G \in \mathcal{G}_{poly} + kv$ if there exists a set $M \subset V$ called modulator, $|M| \leq k$ s.t. $G - M \in \mathcal{G}_{poly}$.

Theorem (Liedloff, M. and Todinca. WG 2015)

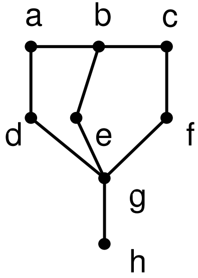
OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t ON $\mathcal{G}_{poly} + kv$ *with parameter k is fixed-parameter tractable, when the modulator is also part of the input.*

The tools

- Tools from [Fomin, Villanger, 2010] and [Fomin, Todinca, Villanger, 2015] for computing a MAXIMUM INDUCED SUBGRAPH OF TREEWIDTH t:
  - Decompositions with minimal separators.
  - Potential maximal cliques
  - Dynamic programming over all minimal tree decompositions of the input graphs, using potential maximal cliques.

- Results [Bodlaender, Kloks, 1996]
  - Given a graph G and a tree decomposition of width at most $k+t$, determine if G has treewidth t in time $\mathcal{O}(f(t+k)n)$, with $f(x) = 2^{\mathcal{O}(x^3 \log(x))}$.

Decomposing with minimal separators



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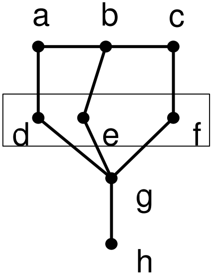

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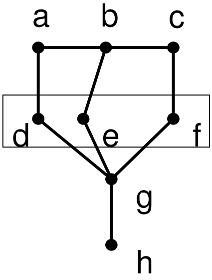

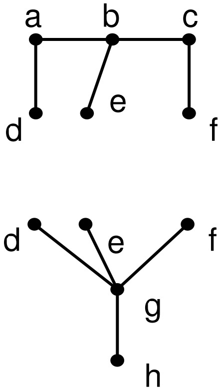

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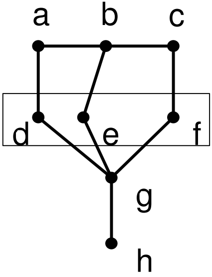

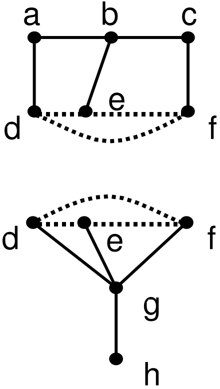

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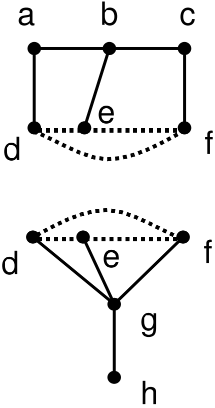

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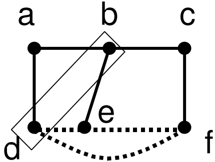

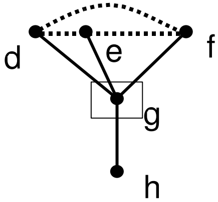

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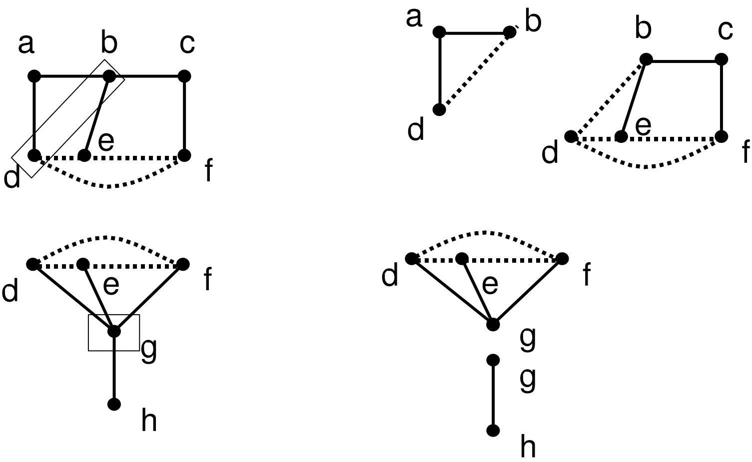

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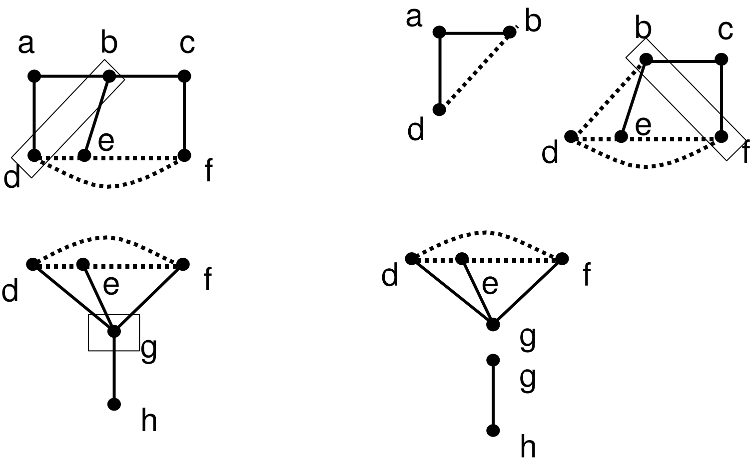

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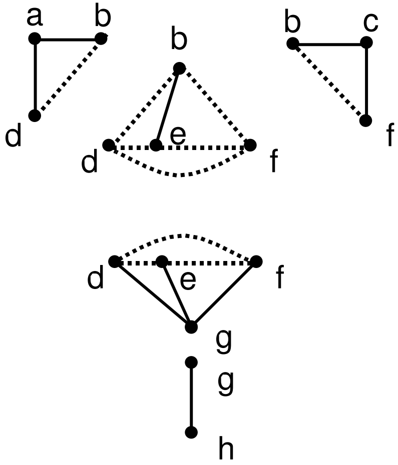
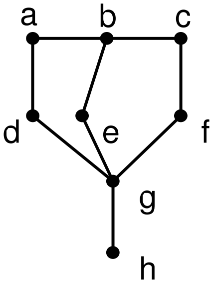
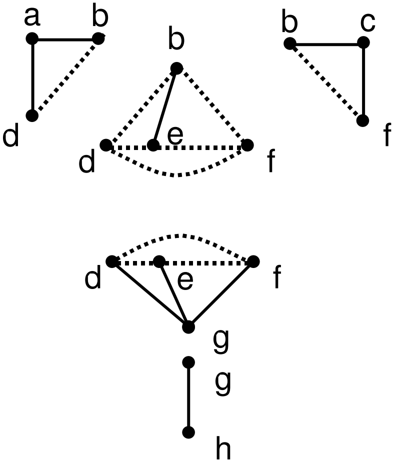


Decomposing with minimal separators





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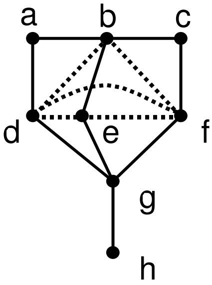

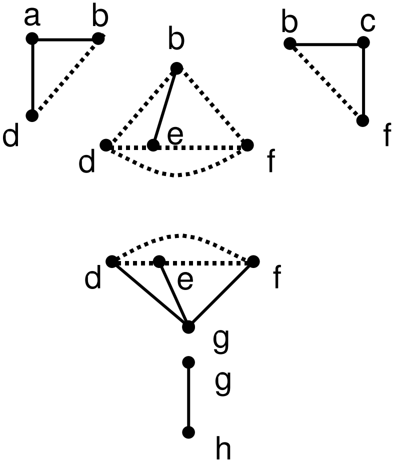

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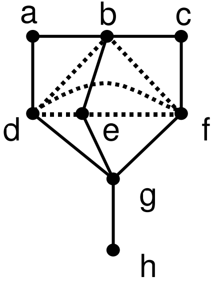
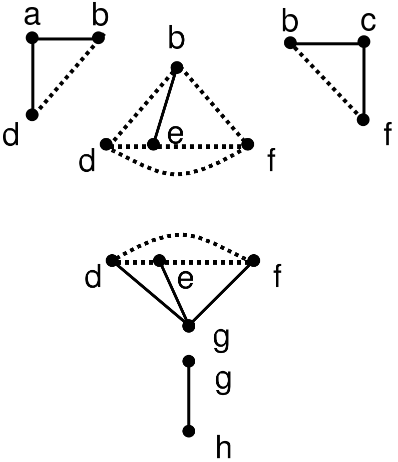




Theorem (Parra, Schaeffler 97)

Decomposing through minimal separators $\rightarrow$ *minimal tree decompositions.*

Potential maximal cliques

Definition

A set of vertices Ω is a potential maximal clique of G if is a maximal clique in some minimal triangulation of G.

Definition

A set of vertices Ω is a potential maximal clique of G if there is a minimal tree decomposition TG of G such that Ω is a bag in TG.

Proposition (Bouchitté, Todinca 2001)

The number of potential maximal cliques is polynomial in the number of minimal separators.

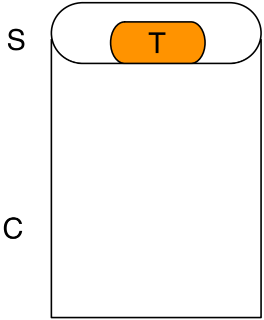
Dynamic programming over minimal separators. . .

MAXIMUM INDUCED SUBGRAPH OF TREEWIDTH t ON $\mathcal{G}_{poly}$

S: minimal separator of G
C: component of $G - S$
T: a subset of S of size $\leq t + 1$

$OPT(S, C, T)$ the size of the largest partial solution $G[F]$ s.t.

- $F \subseteq S \cup C$
- $T = F \cap S$



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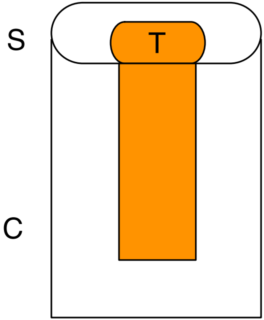

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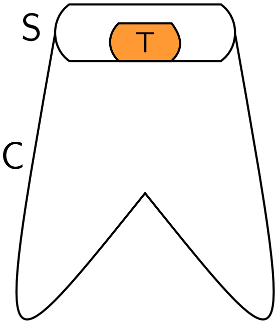

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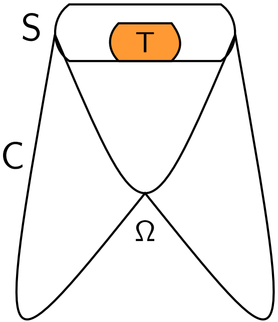

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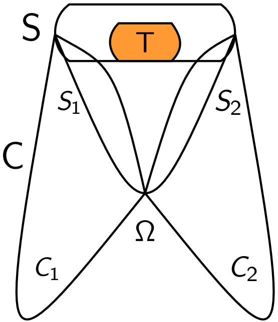

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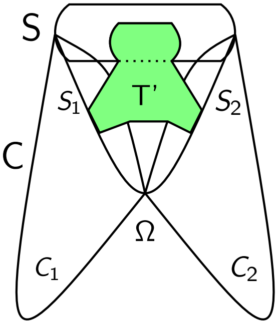

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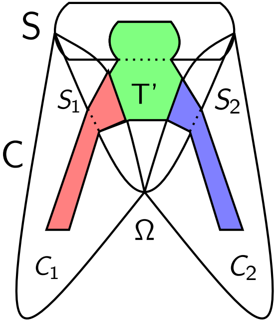

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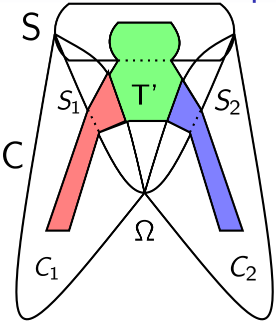

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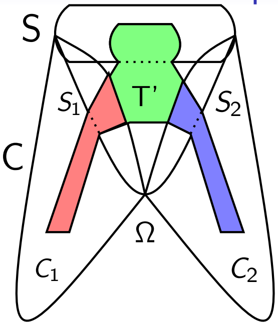

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Running time: $O(n^{t+cst} \cdot \#\text{potential maximal cliques})$

Key lemma [Fomin, Villanger 2010]: we don't miss solutions.

Dynamic programming over minimal separators...

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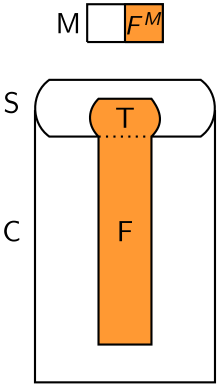

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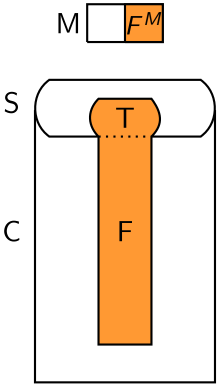

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This way we build a tree decomposition of $G[F]$ of width $\leq t + k'$

[Bodlaender, Kloks, 1996]:

- Algorithm that takes as input a graph with a tree decomposition of width at most $t + k$ and decides if this graph has treewidth t in time $\mathcal{O}(f(t + k)n)$, with $f(x) = 2^{\mathcal{O}(x^3 \log(x))}$

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- *Full set of characteristics*: Set of constant size that *encodes* the decision in a partial solution
- if two partial solutions are *glued*, then the characteristic of the resulting graph can be computed from the characteristics of each part.

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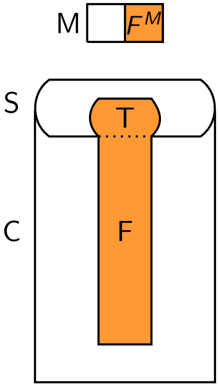

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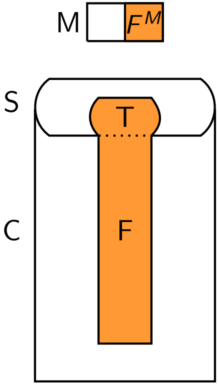

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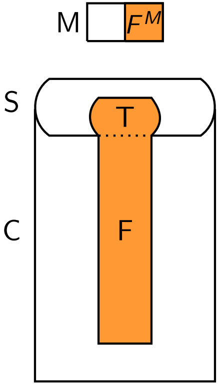

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To sum up

Theorem

OPTIMAL INDUCED SUBGRAPH FOR $\mathcal{P}$ AND t ON $\mathcal{G}_{poly} + kv$ *is solvable in time* $\mathcal{O}(n^t \cdot poly'(n) \cdot f(t+k, \mathcal{P}))$ *when the modulator is also part of the input.*

t	$\mathcal{P}$	f
any	any	tower of exponentials
any	none	$2^{\mathcal{O}((t+k)^3 \log(t+k))}$
0	none	$2^{\mathcal{O}(k)}$
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Thank you!