Complexity and Approximability for Parameterized Constraint Satisfaction Problems

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Definition

(Boolean) Constraint Satisfaction Problem (CSP):

- A set of variables X over $\{0,1\}$.
- A set of constraints ϕ involving positive and negative appearances of the variables (literals).

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Example: CNFSat is a CSP.

$\phi = (\neg x \vee z) \wedge (x \vee y \vee \neg w) \wedge (\neg z \vee w)$

$X = \{x, y, z, w\}$.

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- ...Structural parameterizations

Structural CSP

- Define some graph structure of ϕ.
- Study CSPs for special graph classes.

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Structural CSP

Incidence graph representation of a CSP

- (Unsigned) variables and constraints are represented by vertices;
- a constraint vertex is connected to a variable vertex iff the corresponding constraint involves the corresponding variable.

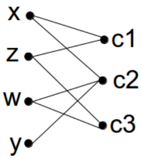

Figure: The incidence graph representation of the previous formula $(\neg x \vee z) \wedge (x \vee y \vee \neg w) \wedge (\neg z \vee w)$.

Structural CSP

- Study CSPs for special graph classes.

Example classes (CNFSAT)

- Low degree: Bounding degree of incidence graph doesn't help (3CNFSAT where every variable appears at most 3 times is NP-complete).

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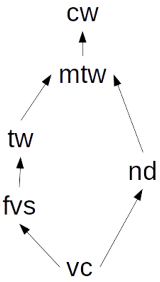
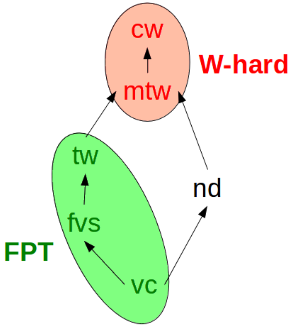
- Low degree: Bounding degree of incidence graph doesn't help (3CNFSAT where every variable appears at most 3 times is NP-complete).
- Acyclicity: Start from the leaves and work your way up (poly-time).

Structural Parameterizations



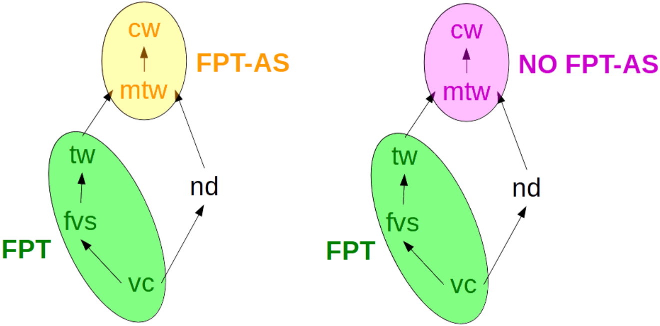
Parameter map: $q \leftarrow p$ (which reads 'q *dominates* p') between two parameters means that q is bounded when p is bounded.

Structural Parameterizations

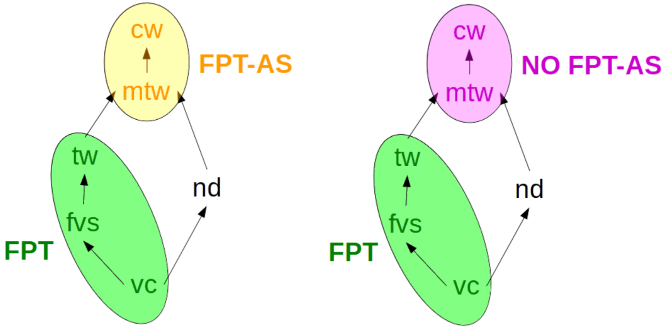


Goal: design algorithms for most dominant parameter (hold downward) and hardness for least dominant (hold upward).

Structural Parameterizations



New approach: study FPT approximations to evade hardness. In this talk we examine the existence of FPT Approximation Schemes.



Definition

FPT Approximation Scheme (FPT-AS): $\forall \epsilon > 0$ there is an $(1-\epsilon)$-approximation algorithm running in time $O(f(\epsilon, k) \cdot \text{poly}(n))$.

Constraints Studied

We explore four natural, well-studied boolean functions which exhibit wildly different behaviors:

1. *or* constraints;

An assignment satisfies such a constraint if at least one literal is made true.

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An assignment satisfies such a constraint if the sum of the literals is odd - or even.

Constraints Studied

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1. *or* constraints;
2. *and* constraints;
3. *parity* constraints;
4. *majority* constraints.

An assignment satisfies such a constraint if at least half of the literals are made true.

Overview of Results

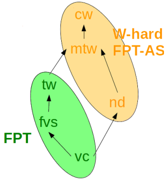


Figure: Diagram for CNFSAT and MAXCNFSAT.

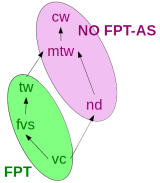


Figure: Diagram for MAXDNFSAT.

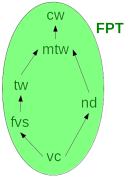


Figure: Diagram for MAXPARITY.

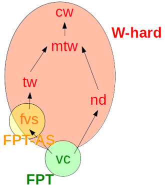
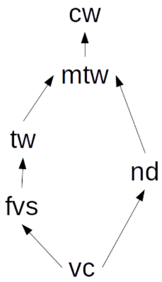
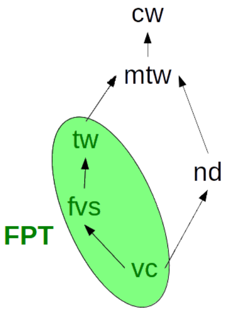


Figure: Diagram for MAJORITY and MAXMAJORITY.


cw
mtw
tw
nd
fvs
vc

Theorem [Szeider 2004]

MaxCNFSat parameterized by incidence treewidth (tw^*) is FPT.



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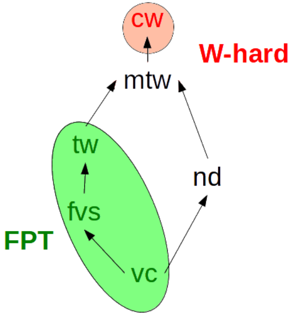

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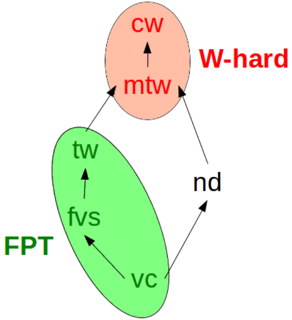

CNFSAT and MAXCNFSAT

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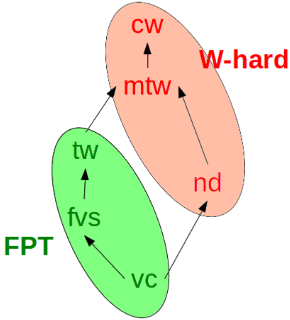
$\rightarrow$ **We extend W[1]-hardness to incidence neighborhood diversity (nd^*).**



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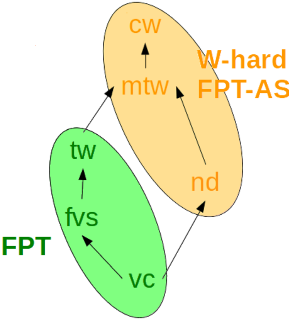
$\rightarrow$ **We extend W[1]-hardness to incidence neighborhood diversity (nd^*).**

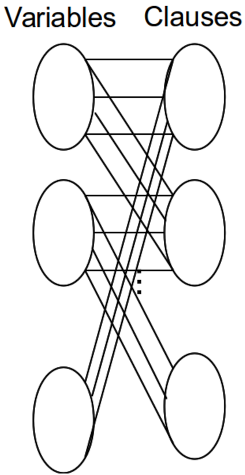
$\rightarrow$ **We also present an FPT-AS for cw^*.**



Neighborhood diversity

Definition [Lampis 2010]

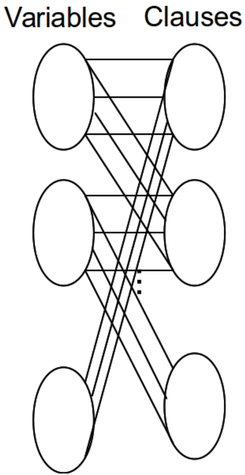
A graph has neighborhood diversity k if its vertices can be separated into k independent sets or cliques where any two vertices in a set have common neighborhood.



Formula ϕ has $\mathrm{nd}^*(G_\phi) \leq k$

There are at most k blocks of variables and clauses, where:

- variables in the same block belong in the same clauses;
- clauses in the same block involve the same variables.



Neighborhood diversity

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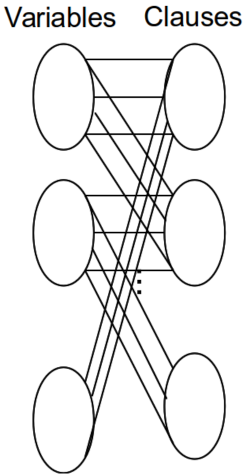
Positive and negative appearances of variables can:

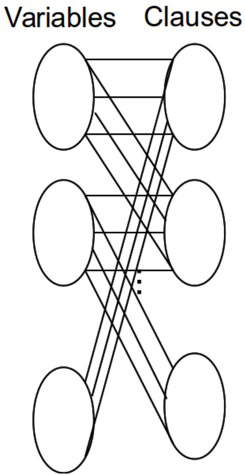
- produce exponentially many clauses;
- encode exponentially many items in binary.



Neighborhood diversity

Theorem

CNFSat *parameterized by the incidence neighborhood diversity* $nd^*(G_\phi)$ *is W[1]-hard.*



On the positive side...

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MaxCNFSat *parameterized by* cw^* *admits an FPT-AS.*

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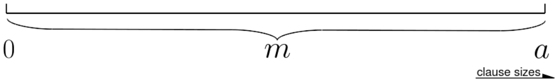
Reminder

FPT-AS (FPT Approximation Scheme) for a maximization problem parameterized by k: $\forall \epsilon > 0$ there exists an $(1 - \epsilon)$-approximation algorithm running in $O(f(\epsilon, k) \cdot \text{poly}(n))$.

An FPT-AS for MAXCNFSAT parameterized by cw*



Arrange the clauses in increasing order of arity (0 to a).

An FPT-AS for MaxCNFSat parameterized by cw*

B: Clauses of arity $\geq D = g(\epsilon)$.
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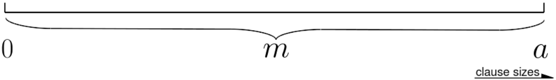
$D = d \cdot \epsilon^4$



Arrange the clauses in increasing order of arity (0 to a).
Split them into big (arity at least $g(\epsilon)$), small (arity at most $g'(\epsilon)$, and medium.
Consider the following cases:

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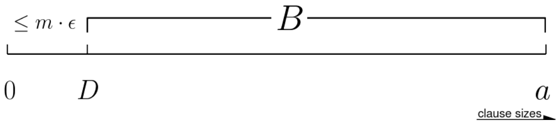

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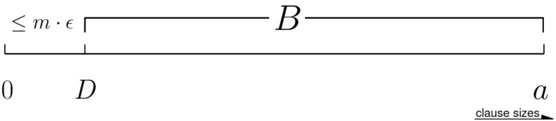

(Almost) all clauses are big:

- ignore small clauses;

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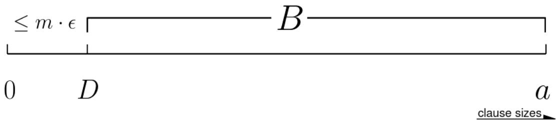

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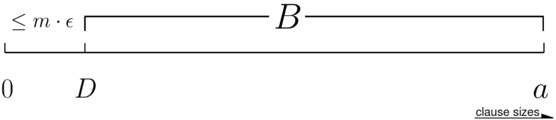

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- Since $m \geq OPT$, $SOL \geq (1-\epsilon')OPT$, for some ϵ' depending on ϵ.

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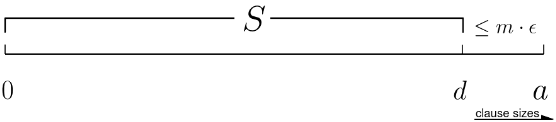

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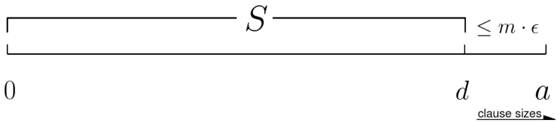

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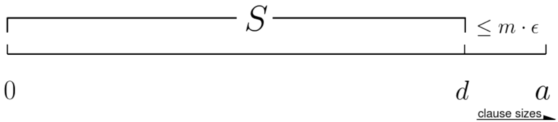

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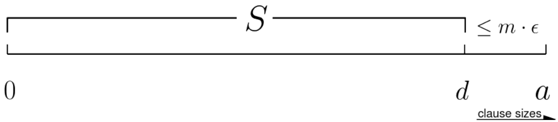

(Almost) all clauses are small:

- ignore large clauses;
- degree on one side of the incidence graph is bounded $\rightarrow$ no large biclique subgraphs;
- By [Gurski, Wanke 2000], the incidence graph has bounded treewidth $\rightarrow$ solve optimally the remaining small clauses;

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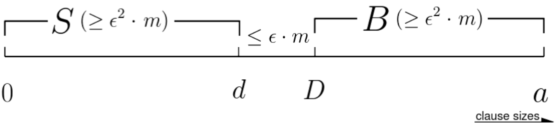

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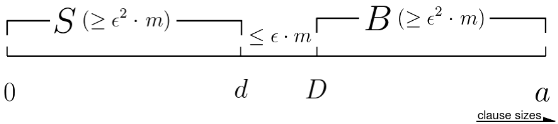

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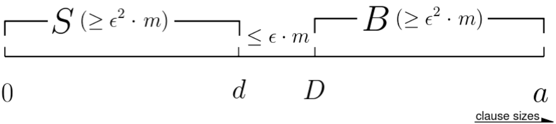

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$\rightarrow \exists y \in V$ that appears $1/\epsilon^2$ more times in B than in S.

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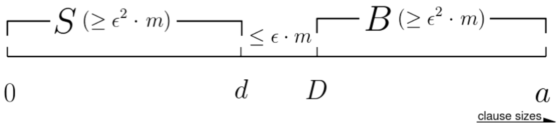

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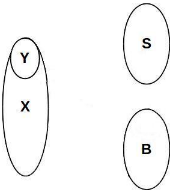

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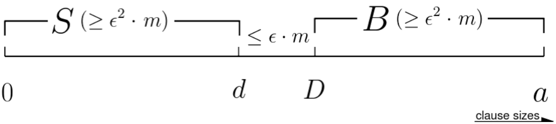

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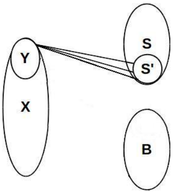

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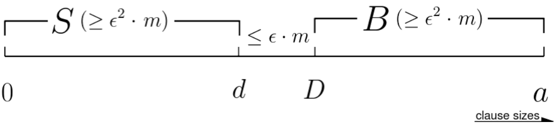

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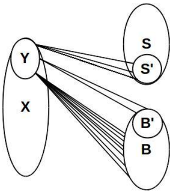

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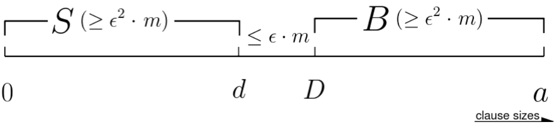

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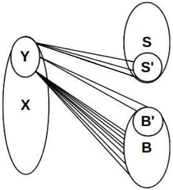

Randomly assigning Y should satisfy whp $\geq (1-\epsilon^2)\cdot(1-2^{-1/\epsilon})$ of $B \setminus B'$, while $S \setminus S'$ can be solved optimally.

Lemma

We can always find a small set M ($|M| \leq \epsilon \cdot m$) of medium-size clauses (arities $d \sim D$).

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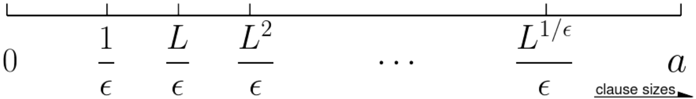

Define $1/\epsilon + 1$ independent intervals of medium-arity clauses (right-left bounds are an $L(= \epsilon^{-4})$-factor apart).

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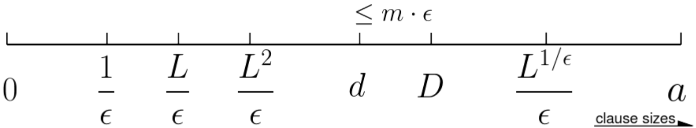

There should be at least one interval $[d, D]$ ($D = L \cdot d$) containing $\leq \epsilon \cdot m$ clauses.

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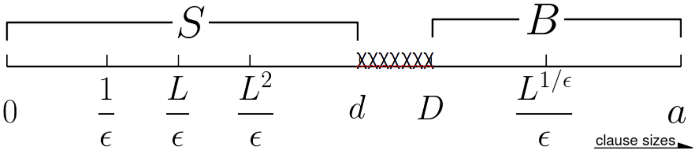

Removing them divides the clauses into small (S) and big (B).

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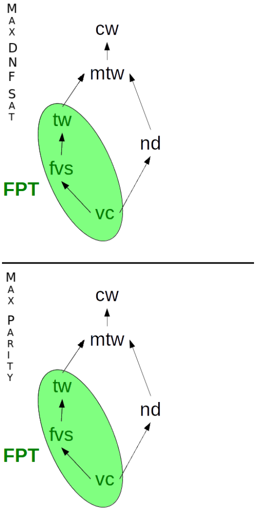

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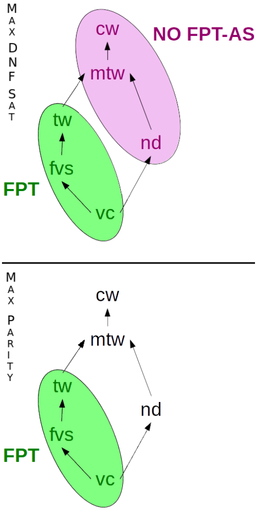

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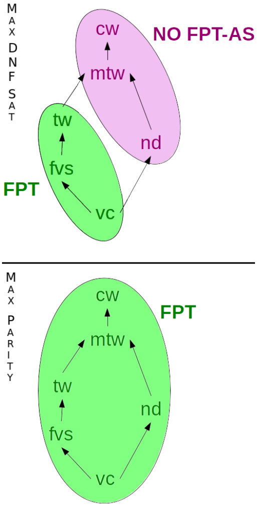

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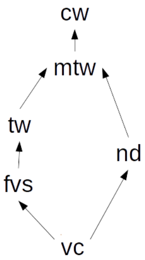

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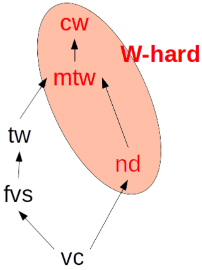

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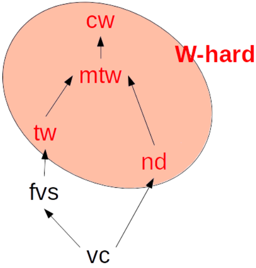

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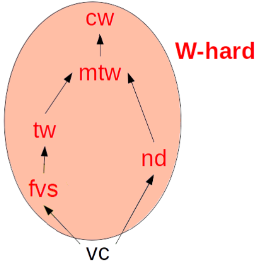

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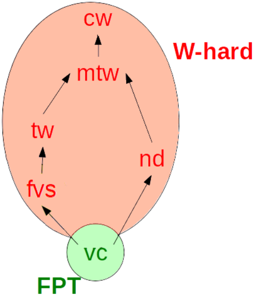

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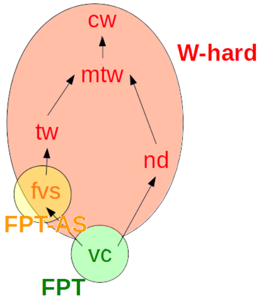

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We can reduce Majority to an ILP with 3^m variables $\rightarrow$ FPT [Lenstra 1983].

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Conclusions

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- Structural parameterizations (incidence graph).
- Provided some of the first **parameterized approximation** results for CSP.
- Complexity-wise, studied CSPs exhibit wildly different behaviors (FPT, W-hard admitting FPT-AS, no FPT-AS unless FPT=W[1]).
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Summary of Results

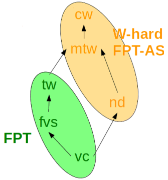
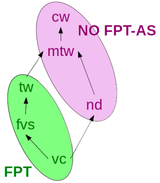


Figure: Diagram for CNFSat and MaxCNFSat.



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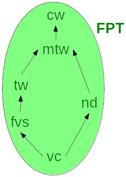
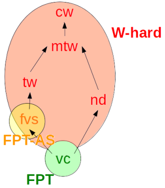


Figure: Diagram for MaxParity.



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Thank you!
Questions?