Cutwidth in Tournament and Applications

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Abstract: In the proof of the graph minor theorem [4], Robertson and Seymour introduced the concept of tree decompositions and their associated width measures. Alternative width measures have been proposed to deal with directed graphs. But so far, the development of a minor theory for directed graphs is still limited. Only recently [1], semi-complete digraphs have been proved to be well-quasi ordered under the *immersion* inclusion relation. The width measure associated to the immersion relation in semi-complete digraph is the *cutwidth* [2,1,3] which is defined as follows. Let $D = (V, E)$ be a digraph and π be an ordering of V. The *cutwidth* of π is

$$\mathsf{cw}(D, \pi) = \max_{0 \leqslant i \leqslant |V|} |E(V_i, V \setminus V_i)|$$

where V_i is the set containing the i first vertices of V in π and $E(V \setminus V_i, V_i)$ denotes the set of arcs from $V \setminus V_i$ to V_i. The *cutwidth* of D is given by

$$\mathsf{cw}(D) = \min_{\pi} \mathsf{cw}(D, \pi)$$

In this talk, we focus on *tournaments*, a subclass of semi-complete digraphs. It is easy to observe that the cutwidth of a transitive tournament is 0. It follows that the well-known Feedback Arc Set in Tournament (FAST) problem can be expressed as the problem of deciding the existence of a subset of at most k arcs whose reversal yields a cutwidth-0 tournament. A natural generalization is to consider the same question with respect to cutwidth-c for a fixed constant c. We call this problem c-Cutwidth Arc Reversal in Tournament (c-CART).

As a consequence of the facts that 1) tournaments are well-quasi ordered under the immersion inclusion relation [2,1] and 2) testing whether a digraph H is included as an immersion in a semi-complete D is FPT [1], the c-CART problem can be solved in non-uniform FPT time. We show that c-CART can be solved in single exponential FPT time. The algorithm follows from the fact that any tournament T such that $\mathsf{cw}(T) \geqslant c$, contains an induced subtournament T' with $O(c^2)$ vertices such that $\mathsf{cw}(T') \geqslant c$. Moreover such a quadratic size obstruction can be found in polynomial time. Finally we observe that these obstructions also lead to single exponential FPT algorithm for the vertex deletion version of the problem.

References

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