Mixed structures in digraphs and completing orientations of partially oriented graphs

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Abstract: This talk has two parts. We first discuss the complexity of problems concerning mixed structures in digraphs. Examples of such problems are

- [4–6] Given a digraph D; does its underlying undirected digraph $UG(D)$ contain two (edge)-disjoint cycles C, W so that C is a directed cycle in D, whereas the edges of W do not have to respect the orientation in D.
- [7, 9] Given a digraph D; does $UG(D)$ have two edge-disjoint spanning trees T_1, T_2 so that T_1 is an out-branching from some root s in D (that is, s can reach every other vertex using only the arcs of T_1), whereas the edges of T_2 do not have to respect the orientation in D.
- [3] Given a digraph D and vertices s, t, u, v of D; does $UG(D)$ contain two (edge)-disjoint paths P_1, P_2 so that P_1 is a directed (s, t)-path in D and P_2 is a path (not necessarily respecting the orientation of arcs of D) from u to v in $UG(D)$.
- [1] Given a digraph D; does $UG(D)$ contain a 2-factor (spanning collection of disjoint cycles) $C_1, \ldots, C_k$ so that C_1 is a directed cycle in D but the other cycles need not be directed cycles in D.
- [8] Mixed multicut: Given a digraph D vertices (terminals) $t_1, t_2, \ldots, t_r$ of D and a mixed pattern graph $M = (\{t_1, t_2, \ldots, t_r\}, \hat{E} \cup \hat{A})$ and a natural number k; Does there exist a set S of at most k arcs of D so that $D' = D - S$ satisfies the following: If $t_it_j \in \hat{E}$, then there is no path between t_i and t_j in $UG(D')$ and if $(t_i, t_j) \in \hat{A}$, then there is no directed (t_i, t_j)-path in D'.

In the second part of the talk we consider orientation completion problems. These can be defined as follows: Let $\mathcal{C}$ be a fixed class of digraphs (e.g. locally semicomplete, acyclic, having a directed cycle factor, strongly connected, locally transitive ...). The **orientation completion problem** for the class $\mathcal{C}$ is to decide, for a given partically oriented graph $P = (V, E \cup A)$, whether we can orient the edges in E so that the resulting digraph $D = (V, \overrightarrow{E} \cup A)$ belongs to the class $\mathcal{C}$. This is a common generalization of the recognition problems for the underlying graphs of digraphs and membership of a digraph class. The problem also has close relations to so-called **representation extension** problems where we are given a partial geometric representation of an induced subgraph of graph and the question is whether we can complete this to a full representation of the whole graph (see e.g. [10, 11]).

We will discuss recent results [2] on the problem when $\mathcal{C}$ is (a subclass of) the class of locally semicomplete digraphs. This includes the problem of orienting a semicomplete digraph as a locally transitive (every in- and out-neighbourhood is transitive) tournament.

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