Polynomial Fixed-Parameter Algorithms:  
A Case Study for Longest Path on Interval Graphs  
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Abstract:
We study the design of fixed-parameter algorithms for problems already known to be solvable in polynomial time. The main motivation is to get more efficient algorithms for problems with unattractive polynomial running times. Here, we focus on a fundamental graph problem: Longest Path; it is NP-hard in general but known to be solvable in $O(n^4)$ time on $n$-vertex interval graphs. We show how to solve Longest Path on Interval Graphs, parameterized by vertex deletion number $k$ to proper interval graphs, in $O(k^9n)$ time. Notably, Longest Path is trivially solvable in linear time on proper interval graphs, and the parameter value $k$ can be approximated up to a factor of 4 in linear time. From a more general perspective, we believe that the idea of using parameterized complexity analysis for polynomial-time solvable problems offers a very fertile ground for future studies for all sorts of algorithmic problems. It may enable a refined understanding of efficiency aspects for polynomial-time solvable problems similarly to what classical parameterized complexity analysis does for NP-hard problems.

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