# 1-Sperner hypergraphs and new characterizations of threshold graphs

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**Abstract:**

We introduce a new class of hypergraphs, the class of 1-Sperner hypergraphs. A hypergraph $\mathcal{H}$ is said to be 1-*Sperner* if every two distinct hyperedges $e, f$ of $\mathcal{H}$ satisfy $\min\{|e \setminus f|, |f \setminus e|\} = 1$. We prove a decomposition theorem for 1-Sperner hypergraphs and examine several of its consequences, including bounds on the size of 1-Sperner hypergraphs and a new, constructive proof of the fact that every 1-Sperner hypergraph is threshold. (A hypergraph $\mathcal{H} = (V, \mathcal{E})$ is said to be *threshold* if there exist a non-negative integer weight function $w : V \to \mathbb{Z}_+$ and a non-negative threshold $t \in \mathbb{Z}_+$ such that for every subset $X \subseteq V$, we have $\sum_{x \in X} w(x) \geq t$ if and only if $X$ contains a hyperedge.)

A hypergraph is said to be *Sperner* (or: *a clutter*) if no hyperedge is contained in another one. Given a graph $G$, the *clique hypergraph of* $G$ is the hypergraph $\mathcal{C}(G)$ with vertex set $V(G)$ in which the hyperedges are exactly the maximal cliques of $G$. The clique hypergraphs of graphs are known to be exactly those Sperner hypergraphs $\mathcal{H}$ that are also *normal* (or: *conformal*), that is, for every set $X \subseteq V(\mathcal{H})$ such that every pair of elements in $X$ is contained in a hyperedge, there exists a hyperedge containing $X$. We show that within the class of normal Sperner hypergraphs, the (generally properly nested) classes of 1-Sperner hypergraphs, of threshold hypergraphs, and of 2-asummable hypergraphs coincide. This yields new characterizations of the class of threshold graphs.

The talk is based on [1].[1]

## References

[1] E. Boros, V. Gurvich, and M. Milanič. 1-Sperner hypergraphs. Manuscript, 2015.

[1]Remark: The manuscript should be soon (hopefully before GROW) posted on arXiv, in which case the reference can be updated accordingly.