Colouring graphs with constraints on local connectivity

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Abstract:

It is well known that deciding if a graph has a proper vertex k-colouring is NP-complete for any fixed k at least 3. On the other hand, if we restrict our attention to graphs with maximum degree at most k, Brooks' theorem implies that it is easy to find a k-colouring, or determine that none exists, in polynomial time. In this talk, we consider several generalisations of the class of graphs with maximum degree at most k — each defined by constraining the local connectivity of some, or all, pairs of vertices — and consider the complexity of k-colouring for graphs in these classes.

A graph has *maximal local connectivity* k (respectively, *maximal local edge-connectivity* k) if no pair of distinct vertices have more than k internally disjoint (respectively, edge-disjoint) paths between them. We characterise the 3-colourable graphs with maximal local edge-connectivity 3, the 3-colourable 3-connected graphs with maximal local connectivity 3, and the k-colourable k-connected graphs with maximal local edge-connectivity k. It follows that there is a polynomial-time algorithm that, for a graph in one of these classes, finds a 3- or k- colouring, or determines that none exists. On the other hand, deciding the k-colourability of minimally k-connected graphs, or deciding the 3-colourability of $(k-1)$-connected graphs with maximal local connectivity k, is shown to be NP-complete.