Automorphism Groups of Geometrically Represented Graphs

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Abstract:

We describe a technique to determine the automorphism group of a geometrically represented graph, by understanding the induced action of the automorphism group on the set of all geometric representations. Each automorphism of a graph can be decomposed into two parts: an automorphism of a representation and a morphism of a representation to another one. We apply this technique to interval graphs, unit interval graphs, permutation graphs, circle graphs and comparability graphs. We show that interval graphs have the same automorphism groups as trees and unit interval graphs the same as disjoint unions of caterpillars. For permutation (which are comparability graphs of the dimension two) and circle graphs, we show their classes of automorphism groups as slightly larger than for trees, and we give their inductive descriptions. On the other hand, we show that any finite group is the automorphism group of a comparability graph with the dimension at most four.

Our approach combines techniques from group theory (group products, homomorphisms, quotients, actions) with computer science data structures (PQ-trees, modular trees, split trees).