Complexity and Approximability for Parameterized CSPs

H. Dell – Saarland University and Cluster of Excellence, Saarbrücken, E. J. Kim – Université Paris Dauphine, M. Lampis – Université Paris Dauphine, V. Mitsou – SZTAKI, Hungarian Academy of Sciences, Budapest, T. Mömke – Saarland University, Saarbrücken.

Abstract:

Constraint Satisfaction Problems (CSPs) play a central role in almost all branches of theoretical computer science. Starting from CNFSAT, the prototypical NP-complete problem, the computational complexity of CSPs has been widely studied from various points of view. In this paper we focus on two aspects of CSP complexity which have mostly been considered separately so far in the literature: parameterized complexity and approximability. We study four standard predicates and contribute some of the first results in this joint area.

Parameterized CSPs. The vast majority of interesting CSPs are NP-hard [3]. This has motivated the study of such problems from a parameterized complexity point of view, and indeed this topic has attracted considerable attention in the literature [1]. In this paper we focus on *structurally* parameterized CSPs, that is, we consider CSPs where the parameter is some measure of the structure of the input instance. The central idea behind this approach is to represent the structure of the CSP using a (hyper-)graph and leverage the powerful tools commonly applied to parameterized graph problems (such as tree decompositions) to solve the CSP.

Approximation. CSPs also play a central role in the theory of (polynomial-time) approximation. In this context we typically consider a CSP as an optimization problem (MAXCSP) where the goal is to find an assignment to the variables that satisfies as many of the constraints as possible. Unfortunately, essentially all non-trivial CSPs are hard to approximate (APX-hard) [2]. This motivates the question of whether we can find cases where efficient approximations are possible.

Results. We consider four different types of CSPs with *or*, *and*, *parity* and *majority* constraints respectively. The new ingredient in our approach is that, in addition to trying to determine which parameters make a CSP FPT or W-hard, we also ask if the optimization versions of W-hard cases can be well-approximated. We show that these basic predicates display a diverse set of behaviors, ranging from being FPT to optimize exactly (parity), to being W-hard but well-approximable (or, majority), to being W-hard and inapproximable (and). Our results indicate that the point of view of approximability considerably enriches the parameterized complexity landscape of CSPs.

References

[1] M. Grohe. The structure of tractable constraint satisfaction problems. In R. Kralovic and P. Urzyczyn, editors, *MFCS 2006, Stará Lesná, Slovakia, August 28-September 1, 2006, Proceedings*, volume 4162 of *Lecture Notes in Computer Science*, pages 58–72. Springer, 2006.

[2] S. Khanna, M. Sudan, and D. P. Williamson. A complete classification of the approximability of maximization problems derived from boolean constraint satisfaction. In F. T. Leighton and P. W. Shor, editors, *Proceedings of the Twenty-Ninth Annual ACM Symposium on the Theory of Computing, El Paso, Texas, USA, May 4-6, 1997*, pages 11–20. ACM, 1997.

[3] T. J. Schaefer. The complexity of satisfiability problems. In R. J. Lipton, W. A. Burkhard, W. J. Savitch, E. P. Friedman, and A. V. Aho, editors, *Proceedings of the 10th Annual ACM Symposium on Theory of Computing, May 1-3, 1978, San Diego, California, USA*, pages 216–226. ACM, 1978.