Erdös-Pósa Property of planar-H-minor models with prescribed vertex sets

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Abstract:

Robertson and Seymour [5] proved that for every planar graph H, the class of all H-expansions, which are the graphs can be contracted to H, has the Erdös-Pósa property, and this does not hold for non-planar graphs. We generalize this result for disjoint H-expansions containing vertices from two sets among several prescribed vertex sets. This generalization is motivated from Mader's $\mathcal{S}$-path theorem [2].

For a graph G and a set $\mathcal{Z} = \{Z_i \subseteq V(G) : 1 \leq i \leq m\}$ with $m \geq 2$, an H-expansion F in G is *$\mathcal{Z}$-connecting* if $V(F) \cap Z_i \neq \emptyset$ and $V(F) \cap Z_j \neq \emptyset$ for some $i \neq j$. We show that $\mathcal{Z}$-connecting H-expansions have the Erdös-Pósa property: for a positive integer k and a planar graph H, there exists a function $f(k, H)$ such that if G is a graph and $\mathcal{Z} = \{Z_i \subseteq V(G) : 1 \leq i \leq m\}$ with $m \geq 2$, then either

1. G contains k pairwise vertex-disjoint $\mathcal{Z}$-connecting H-expansions, or
2. there is a vertex subset T of size at most $f(k, H)$ in G such that $G \backslash T$ contains no $\mathcal{Z}$-connecting H-expansions.

We point out that the function f does not depend on the number m of prescribed vertex sets. It implies the original theorem by Robertson and Seymour by taking $Z_1 = Z_2 = V(G)$, and further implies that for a vertex set $S \subseteq V(G)$, H-expansions intersecting S also have the Erdös-Pósa property by taking $Z_1 = S$ and $Z_2 = V(G)$, which is similar to the Erdös-Pósa property of cycles intersecting S [1, 4]. To prove it, we use the Rooted Grid Minor theorem developed by Marx, Seymour, and Wollan [3], and mainly develop a procedure to find a vertex in a graph of large tree-width, called an irrelavent vertex, whose deletion preserves the minimum size of a packing set for $\mathcal{Z}$-connecting H-expansions. We also give an example showing that if the number of required intersecting sets is more than two, then the Erdös-Pósa property does not hold. This is joint work with Dániel Marx.

References

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